# Ship Hull Form Optimization Design for KCS Considering Uncertainty of Ship Speed

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## ABSTRACT

At present, most of ship hull form optimization design is based on deterministic parameters. However, in the actual engineering problem, there are many unavoidable uncertain factors. For instance, the ship speed may vary around the design speed due to wind, waves, flow and other environmental or human factors while sailing. The optimal ship based on deterministic parameters may fail to reach the predetermined target while parameters change. An uncertainty optimization method based on stochastic programming is applied to reduce the resistance of KCS considering uncertainty of ship speed in this paper. The whole optimization process is based on an in-house ship hull form optimization solver, OPTShip-SJTU. Comparison and analysis of the resistance and flow field between initial ship and optimal ship validates the rationality and effectiveness of uncertainty optimization based on stochastic programming in ship hull form optimization design field.

KEY WORDS: Ship hull form optimization; Uncertainty optimization; Stochastic programming; OPTShip-SJTU.

## INTRODUCTION

As a core part of ship overall design, ship hull form design based on simulation-based design (SBD) technology, which is developed by combining the optimization technique and computational fluid dynamics (CFD) technique, plays a more and more important role in ship design field. There are three crucial elements for SBD optimization design process, automatic modification of ship hull geometry, high precision simulation method, and advanced optimization algorithms.

However, a large number of alternatives should be evaluated during the optimization process. Establishing the approximation model as a surrogate is an efficient way to reduce calculation burden. Despite the high efficiency, approximation models have the disadvantage at the same time: the establishment of approximation models needs amounts of precision results as input, and output results are sensitive to internal

parameters, so inevitably, error of output will occur due to some uncontrollable causes, which is expressed as uncertainty. Although this error or uncertainty has a small value in most cases, large deviation of the whole system can also be generated by continuous iterative computation. Therefore, it has an important theoretical and practical significance for considering the uncertainty of approximate model.

Two uncertainty optimization methods are broadly applied in practical engineering: interval programming, stochastic programming. Interval programming only needs interval number, with the upper and lower bounds of the uncertain parameter, and more details can be found in Hou *et al.* (2017). By comparison, stochastic programming achieves optimum solution via random variable, with given probability density distribution, which reflects the actual situation better. Thus uncertainty optimization method based on stochastic programming, such as robust design optimization (RDO) and reliability-based design optimization (RBDO), has been widely used in many engineering fields, such as airplane component design (Steenackers *et al.* 2009), V6 engine design (Wang *et al.* 2009), and structural design (Lagaros *et al.* 2007), but with little use in ship hull form design field.

An uncertainty optimization design based on stochastic programming for minimum resistance of KCS considering the uncertainty of ship speed is proposed in this paper. The whole optimization process is based on an in-house ship hull form optimization solver, OPTShip-SJTU. The geometry of hull form is modified by the free form deformation (FFD) method, and the main deformed areas are bow and stern. An efficient and robust potential theory, Neumann-Michell (NM) theory is integrated in the optimization process to evaluate the objective functions for ship hydrodynamics. The approximation model for the total resistance coefficient is constructed by Kriging method based on the samples produced by optimized Latin hypercube sampling (OLHS) method to shorten computation time. In the optimization process, Froude number is taken as a random variable with given probability density distribution (PDF), and a stochastic programming based on genetic algorithm (GA) is applied to solve this uncertainty optimization problem. Comparison and analysis of the resistance and flow field between initial and optimal hull validate the effectiveness of uncertainty optimization design for the



Fig. 1 The flow chart of the iterative optimization process

related cases. The flow chart of the iterative optimization process is illustrated in Fig. 1.

# UNCERTAINTY OPTIMIZATION WITH STOCHASTIC PROGRAMMING

Uncertainty optimization problem (Hou, 2017) can be described as the following equation:

$$\begin{cases} \min f(x,u) \\ s.t. g(x,u) \le 0 \end{cases}$$
(1)

where x is a vector of design variables, u is a vector of uncertainty, f is the objective function and g is the restrictive function.

In stochastic programming, a new objective function  $F_f(x)$  is defined as following, using a weighted-sum approach (Shimakage *et al.* 2011):

$$F_f(x) = \alpha \mu_f(x) + (1 - \alpha)\sigma_f(x)$$
<sup>(2)</sup>

where  $\mu_f$  and  $\sigma_f$  are the expected value (EV) and standard deviation (SD) of the response function f at the design point x with the uncertainty u, where  $\alpha$  is a weighting parameter,  $0 \le \alpha \le 1$ , that can be adjusted for the relative importance of EV and SD for the particular application.

The following equations can be used to calculate EV and SD of the response function f that are needed for Eq. 2:

$$\mu_f(x) = \int f(x,\xi) p(\xi) d\xi \tag{3}$$

$$\sigma_f(x) = \sqrt{\int (f(x,\xi))^2 p(\xi) d\xi - (\mu_f(x))^2}$$
(4)

In Eqs. 3~4,  $\xi$  is the random variable for uncertainty u, and p( $\xi$ ) is the probability distribution function (PDF), which can be extracted from historical data. Once PDF obtained, the uncertainty optimization problem can be transformed into deterministic optimization problem and



Fig. 2 An application of FFD method to modify a ship bow.

solved. In this paper, the uncertainty of ship speed will be taken as an example.

#### MODIFICATION OF HULL GEOMETRY

Appropriate and effective surface modification methods are critical to optimization process. On the one hand, these techniques should modify hull forms efficiently and ensure the rationality of the new hull surface, on the other hand, the number of variables involved in these methods should keep as low as possible—too many design variables will increase the complexity of the problem and lead to vast computational cost. In this paper, FFD method is applied to modify the geometry of hull form.

FFD technique, proposed by Sederberg and Parry (1986) based on trivariate Bernstein polynomials, is utilized to perform the deformation of solid geometric models in a free-form manner. In this method, the objects to be deformed are embedded into a plastic parallelepiped, and then these objects are deformed along with it. The modification of hull form is defined and controlled by using a few control nodes, and the displacements of them are utilized as design variables by optimizer. An application of FFD method to modify a ship bow is shown as Fig. 2 (Wu *et al.* 2017).

#### NEUMANN-MICHELL THEORY

A practical simulation tool is one of the main components for hull form optimization procedure. The optimizer is guided by the evaluating results involved thousands of alternatives toward the improved solutions, accordingly, both accuracy and efficiency are important for this tool. In this study, Neumann-Michell (NM) theory is employed to evaluate the drag of a ship hull.

When a ship steadily advances at constant speed along a straight path in calm water of effectively infinite depth and lateral extent, the wave drag related to the waves generated by the advancing ship hull is of considerable practical importance because drag is a critical and dominant hydrodynamic factor for ship design. The Neumann-Michell (NM) theory is an efficient potential flow theory used to predict the ship waves. In this theory, both of the surface tension and the free surface nonlinearities are ignored for the practical goal, and the viscosity effect is estimated by considering the turbulent viscous boundary layer on a flat plate. This theory is the modification of the Neumann-Kelvin (NK) theory based on a consistent linear flow model. The main difference between the two theories is that the line integral around the ship waterline that occurs in the classical NK boundary-integral flow representation is eliminated in the NM theory, so the NM theory expresses the flow about a steadily advancing ship hull in terms of a surface integral over the ship hull surface.

The Neumann-Michell potential representation and more details of this

theory are provided by Noblesse *et al.* (2013). This theory can yield realistic predictions of wave drags at low computational cost. Due to the simplicity and fast computation, the similar potential theory has been used for hull-form optimization (Kim *et al.* 2009; Kim *et al.* 2013).

## **OPTIMIZATION MODULE**

Optimization module plays an important role in the optimization tool. The aim of this module is to minimize objective functions during the optimization process. A global optimal algorithm, genetic algorithm (GA), is adopted in present study. However, the computation associated with global optimal algorithm and simulation codes is very expensive, so a statistical approximation model with DOE method is employed to reduce the computational cost.

#### **Design of Experiment**

Constructing approximation model (or metamodels) based on computer experiments is becoming widely used in engineering to reduce the computational cost. In order to improve the space-filling property and computational efficiency in sampling, various design methods has been proposed, such as Latin hypercube design (LHD) (Simpson *et al.* 2001).

In this work, optimal Latin hypercube sample method (Jin *et al.* 2005), a modified Latin hypercube design, is used for sampling. An application of optimal Latin hypercube design with two factors and nine design points is illustrated in Fig. 3. Fig. 3(a) shows the standard orthogonal array and Fig. 3(b) shows the random Latin hypercube design matrix. In Fig. 3(c), the optimal Latin hypercube design matrix is displayed, and the design points cover all levels of each factor as well as spread evenly within the design space.



Fig. 3 Three types of experimental design method

## **Mathematics of Kriging Model**

Kriging model (Simpson, 1998) is developed from mining and geostatistical applications involving spatially and temporally correlated data. This model combines a global model and a local component:

$$y(x) = f(x) + z(x) \tag{5}$$

where y(x) is the unknown function of interest, f(x) is a known approximation function of x, and z(x) is the realization of a stochastic process with mean zero, variance  $\hat{\sigma}^2$ , and non-zero covariance.

The kriging predictor is given by:

$$\hat{y} = \hat{\beta} + \mathbf{r}^{T}(x)\mathbf{R}^{-1}(\mathbf{y} - \mathbf{f}\hat{\beta})$$
(6)

where **y** is an n<sub>s</sub>-dimensional vector that contains the sample values of the response; **R** is the correlation matrix; **f** is a column vector of length n<sub>s</sub> that is filled with ones when **f** is taken as a constant;  $\mathbf{r}^{T}(x)$  is the

correlation vector of length  $n_s$  between an untried x and the sampled data points  $\{x^{(1)}, x^{(2)}, ..., x^{(n_s)}\}$  and is expressed as:

$$\mathbf{r}^{T}(x) = [R(x, x^{(1)}), R(x, x^{(2)}), ..., R(x, x^{(n_{s})})]^{T}$$
(7)

Additionally, the Gaussian correlation function is employed in this work:

$$R(x^{i}, x^{j}) = \exp[-\sum_{k=1}^{n_{dv}} \theta_{k} \left| x_{k}^{i} - x_{k}^{j} \right|^{2}]$$
(8)

In Eq. 6,  $\hat{\beta}$  is estimated as:

$$\hat{\boldsymbol{\beta}} = (\mathbf{f}^T \mathbf{R}^{-1} \mathbf{f})^{-1} \mathbf{f}^T \mathbf{R}^{-1} \mathbf{y}$$
(9)

The estimate of the variance  $\hat{\sigma}^2$ , between the underlying global model  $\hat{\beta}$  and **y** is estimated using Eq. 10:

$$\hat{\sigma}^2 = \left[ (\mathbf{y} - \mathbf{f}\hat{\beta})^T R^{-1} (\mathbf{y} - \mathbf{f}\hat{\beta}) \right] / n_s$$
(10)

where f(x) is assumed to be the constant  $\hat{\beta}$ . The maximum likelihood estimates for the  $\theta_k$  in Eq. 8 used to fit a kriging model are obtained by solving Eq. 11:

$$\max_{\theta_k > 0} \Phi(\theta_k) = -[n_s \ln(\hat{\sigma}^2) + \ln |\mathbf{R}|]/2$$
(11)

## **Optimization Algorithm**

In this paper, genetic algorithm (GA) is adopted to drive the optimization procedure, which is successfully applied in many engineering fields. In genetic algorithm, each individual represent as a potential solution to problems. After generating a population of individuals, they are tested for fitness and scored by a certain fitness value. The more outstanding an individual is, the easier it is to make the heritage of their "genes" (encoded optimization variables) inherited by the next generation. The dominant individuals in the parental population produce the same number of offspring via variation or random crosses. After several generations, the algorithm converges to the optimal individual, which means the optimal solution.

## DEFINITION OF THE OPTIMIZATION PROBLEM

## **Initial Hull Form**

The geometry of the initial model is presented in Fig. 4 and the principal dimensions of KCS in Table 1. The optimization is performed for the model scale.

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Principal	Full-scale ship	Ship model
L <sub>pp</sub> /m	230	7.28
L <sub>wl</sub> /m	232.5	7.36
B/m	32.2	1.019
D/m	19	0.6013
T/m	10.8	0.3418
Cb	0.651	0.651



Fig. 4 Initial model of KCS



Fig. 5 Deformed areas of bow and stern

Table 2. Variable range of design variables

Design variables	Min	Max
X1 (bow-x axis)	-0.01	0.01
Y1 (bow-y axis)	-0.01	0.01
Z1 (bow-z axis)	-0.02	0.02
Y2 (stern-y axis)	-0.02	0.02

#### **Design Variables and Geometrical Constraints**

In FFD method, the modification of hull form is defined and controlled by using a few control nodes (red nodes in Fig. 5), and the displacements of them are utilized as design variables by optimizer. In this paper, the main deformed areas are bow and stern, shown in Fig. 5. There are totally 4 design variables, X1, Y1, Z1, Y2; the first three respectively control the modification along x, y, z axis of bow, and the last one control the modification of hull form, the variable range of design variables are restricted as in Table 2.

Additionally, some geometric constraints are imposed on the design variables. Thereinto,  $L_{pp}$ , D, B are fixed, and the change of displacement should be within  $\pm 0.4\%$ .

### **Objective Function Considering Uncertainty of Ship Speed**

In this paper, minimum total resistance coefficient of KCS Ct is taken as the response function f, and the uncertainty of ship speed (Fr) is taken as the random variable  $\xi$ . Therefore, the objective function F(x) can be calculated in following equations:

$$F_{Ct}(x) = \alpha \mu_{Ct}(x) + (1 - \alpha)\sigma_{Ct}(x)$$
(12)

$$\mu_{Ct}(x) = \int Ct(x, Fr) p(Fr) dFr$$
(13)

$$\sigma_{Ct}(x) = \sqrt{\int (Ct(x, Fr))^2 p(Fr) dFr - (\mu_{Ct}(x))^2}$$
(14)

The probability distribution function of Fr can be extracted from historical sailing data. But in this paper we simplify PDF based on following assumption:

For a selected design ship speed, the actual ship speed follows a normal distribution about  $V_{\rm design};$  that is,

$$V_{actual} = V_{design} + \nu \tag{15}$$

where  $V_{actual}$  is the actual ship speed and v is a normally distributed random variable with mean zero. This can be used to model the situation that the intended ship speed is  $V_{design}$ , but due to sea conditions the actual ship speed may vary. For this analysis, Fr follows a normal distribution with mean of 0.26 ( $\mu$ ) and standard deviation of 0.026 ( $\sigma$ ). And the normally distributed probability distribution function (PDF) can be used in Eqs. 13~14, which is integrated over  $\pm 3\sigma$  to cover approximately 99.74% of the area under the normal curve. In addition,  $\alpha$  in Eq. 12 is taken as 0.6 due to slight importance of EV in this problem.

## OPTIMAL RESULTS AND ANALYSIS

In this paper, OLHS method is applied to generate a set consisting of 32 sample points, which is used to construct the approximate model via kriging method. Before the optimization, the fidelity of surrogate models needs to be validated, generally by the cross-validation method (Fig. 6). In the cross validation, each sample point is evaluated from the Kriging surrogate model that is constructed by the other 31 sample points. It can be observed that the objective function values estimated by the surrogate model ( $f_{obj}^E$ ) show a good agreement with these values directly evaluated

by the NM theory ( $f_{abj}^{C}$ ). Genetic algorithm obtain the optimal solution after iterations of 50 generations with the population size of 50 (Fig. 7).



Fig. 6 Cross-validation result of surrogate model



Fig. 7 Iterations and convergence of genetic algorithm

Table 3. Comparisons of Ct between	initial hull	and	optimal	hull	in a
range of Fr with $\pm 3\sigma$					

	Ct (*1	Reduction	
Fr	Initial	Optimal	(%)
0.1820	3.58073	3.45870	3.41
0.2015	3.52208	3.42446	2.77
0.2210	3.48480	3.41471	2.01
0.2405	3.50281	3.44818	1.56
0.2600	3.73334	3.68576	1.27
0.2795	4.43083	4.32536	2.38
0.2990	5.03440	4.95050	1.67
0.3185	5.29945	5.18770	2.11
0.3380	5.35514	5.27137	1.56

Table 4. Comparison of EV and SD of Ct between initial and optimal hull considering uncertainty of ship speed

	EV	SD
Fr	0.260	0.026
Ct of initial hull	3.966E-3	5.839E-4
Ct of optimal hull	3.896E-3	5.668E-4
Reduction (%)	1.785	2.928

Table 3 shows the numerical predictions of total resistance and the corresponding reductions for initial and optimal hulls in a range of Fr with  $\pm$  3 $\sigma$ . Fig. 9 illustrates the resistance reductions where the black solid line represents the initial hull and the red dotted line represents the optimal hull. It can be found that the optimal hull performs better while ship speed vibrates around design ship speed, and the expected value (EV) and standard deviation (SD) of total resistance coefficient are listed in Table 4. However, it seems confusing that Table 3 shows the reduction in Ct at the design speed is the smallest, which should be the largest among the speeds with current PDF. We think the reason may be that the initial hull performs better at the design speed than other speeds, so abundant and diverse modification of hull geometry should be applied to the optimization to get satisfactory result.



Fig. 8 Comparison of transverse and longitudinal cross-section curve between initial hull and optimal hull



Fig. 9 Comparisons of Ct between initial hull and optimal hull in a range of Fr with  $\pm$   $3\sigma$ 

Fig. 8 shows the comparison of transverse and longitudinal cross-section curve between initial and optimal hulls. It can be obviously observed that bow and stern both become thinner, besides bow has a tendency to elongate and upwarp. Additionally, the reduction of displacement is 0.34%. Figs. 10~11 depict the comparison of wave pattern and pressure distribution at design ship speed between initial and optimal hulls. Reduction of bow waves can be clearly observed via wave height contours in Fig. 10 and decrease of bow pressure can be found in Fig. 11.





Fig. 10 Comparison of wave pattern between initial hull and optimal hull (Fr=0.26)

Fig. 11 Comparison of pressure distribution between initial hull and optimal hull (Fr=0.26)

## CONCLUSIONS

An uncertainty optimization method based on stochastic programming is applied to reduce the resistance of KCS considering uncertainty of ship speed based on an in-house ship hull form optimization solver, OPTShip-SJTU. For a selected design ship speed, the actual ship speed follows a normal distribution about the design ship speed. Based on this assumption, Froude number is taken as a random variable with given probability density distribution (PDF). During the procedure of optimization, the region of bow and stern is deformed with free-form deformation (FFD) method. The modification techniques are sufficiently flexible to generate a series of realistic alternative hull forms with a few number of design variables involved. A practical simulation tool based on the Neumann-Michell (NM) theory is implemented in the hydrodynamic performance evaluation module to predict the resistance. Optimized Latin hypercube sampling (OLHS) method and kriging model have been employed here to establish the relationship between the objective functions and the design variables to decrease computational effort. The optimizer based on genetic algorithm (GA) obtain the optimal hull eventually.

Comparisons and analysis of transverse and longitudinal cross-section curve, resistance, wave pattern and pressure distribution between the initial and optimal hulls indicate that the optimal hull performs better than initial hull while ship speed vibrates around design ship speed. The result validates the rationality and effectiveness of uncertainty optimization based on stochastic programming in ship hull form optimization design field. And this paper could provide information and reference for ship hull form optimization with more complicated uncertainty.

The future work will focus on applying abundant and diverse modification of hull geometry to the optimization to get better optimization effect. Otherwise, we will optimize Rt instead of Ct, so that the wetted surface area can be automatically taken into consideration.

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